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Thurstan College – Colombo 07

13 ශ්‍රේණි පරීක්ෂණය, 2020 සැප්තැම්බර්

Grade 13 Test, September 2020

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Combined Mathematics I

10 S I

Part B

- **Answer five questions only.**

11' (a) α and β are the roots of the quadratic equation $ax^2+bx+c=0$.

i) Show that $\left(\alpha+\frac{1}{\alpha}\right)^2+\left(\beta+\frac{1}{\beta}\right)^2=\frac{b^2(a^2+c^2)-2ac(a-c)^2}{a^2c^2}$

ii) Show that $acy^2+b(a+c)y+(a-c)^2+b^2=0$ when $y=x+\frac{1}{x}$

Hence obtain the result (i)

(b) i) If $(x-a)^2$ is a factor the function $f(x)$ show that $(x-a)$ is a factor of $f'(x)$ $\left(\frac{df(x)}{dx}\right)=f'(x)'$

ii) Let $f(x)=x^4-px^3-11x^2+4(p+1)x+q \quad p,q\in z^+$

$(x+2)$ is a factor of $f(x)$, and $f(x)$ is a square of a quadratic expression. Find the values of p and q and then obtain the solution of $f(x)=0$.

12. i) Write down the expansion of $(1+x)^n$.

By using the binomial expansion, Show that $(2)^{3n+2}-28n-7 \quad n\in z^+$ is divisible by 49.

ii) Fourteen teams are qualified to participate for T - 20 cricket world cup tournament in 2021. All teams are divided into two groups A and B. Every team should play with every other team. The best three teams of both A and B groups are qualified for the second round. In the second round also. every team should play with every other team. The best four teams are selected for the Semifinal. In the Semifinal round, also each team plays with every other team. The best two teams will be qualified for the final round. There are three matches in the final round. Find the minimum number of matches that should conduct for the T-20 Cricket Series in 2021.

iii) a) Let $1 \equiv A(x+2)(x+3)+B(x+1)(x+3)+C(x+1)(x+2),\ x \in R$ Find the constants A, B, C hence express $\dfrac{1}{(x+1)(x+2)(x+3)}$ as partial fractions.

b) Let $U_r = \dfrac{1}{(r+1)(r+2)(r+3)}$ for $r \in Z^+$

Using the part (a) determined $f(r)$ and λ such that $U_r = \lambda[f(r)-f(r+1)]$

Hence Find $\sum_{r=1}^{n} U_r$.

13. i) Let $A=\begin{pmatrix} a & 2a-7 \\ 3 & a+2 \end{pmatrix}$ $a \in R$

If $|A| = \begin{vmatrix} a & (2a-7) \\ 3 & a+2 \end{vmatrix} = 17$, then find a.

Show that $A^2 - 6A + 17I = 0$. Hence find A^{-1}, where I is a matrix of Order 2.

Show that for any matrix $(B^T)^{-1} = (B^{-1})^T$.

Hence write down the matrix $(A^T)^{-1}$

Find the solution for the simultaneous equation by using matrices.

$2x - 3y = 3$

$3x + 4y = 13$

ii) Show that $z\bar{z} = |z|^2$, where z is a complex number.

(a) Hence obtain the results

$|z_1 z_2| = |z_1||z_2|$

$\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$

(b) z is a complex number such that $|z-\sqrt{2}i| = |z+\sqrt{2}i|$ and

$\text{Arg}|z+\sqrt{2}| = \pi/2$

Show that z = $\sqrt{2}(-1+i)$.

By using Moivre's theorem, express $3z^2$, z^4 in the polar form, and draw in an Argand diagram.

$\left|\dfrac{z^2(z^2+3)}{(z+\sqrt{2}}\right| = 10\sqrt{2}$ and $\left(\dfrac{z^2(z^2+3)}{(z+\sqrt{2}}\right) = \tan^{-1} 3/4 + \pi/2$

14) (a) i) If $y = x^x + 2^x$, find $\dfrac{dy}{dx}$

ii) If $x = a\cos t$ and $y = a\sin t$ show that

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+1=0$$

(b) Let $f(x)=\dfrac{x^2-2}{(x-1)^2}:x\neq 1$ Sketch the graph of y = f(x) indicating the point of inflection asymptotes and turning points.

(c) A right-angle triangle with a maximum perimeter should be drawn inside of a circle of radius a. Hypotenuses of the triangle should be the diameter of the circle. Find the length of three sides of the triangle, and the maximum perimeter.

15) (a) Find the values of the constants A, B, and C such that

$$A(1-x+x^2;+(Bx+C)(1+x)=1$$

Hence find $\displaystyle\int\frac{dx}{1+x^3}$

(b) Using the substitution $\tan x/_2 = t$ show that $\displaystyle\int_0^{\pi/2}\frac{dx}{1-\sin x\cos\lambda}=(\pi-\lambda)\cos ee\lambda$

where $(o<\lambda<\pi)$

(c) By using integration by parts, Find $\int e^{-ax}\cos bx\,dx$ $\quad a>o$

(d) By using $\displaystyle\int_0^a f(x)dx=\int_0^a f(a-x)dx$ show that $\displaystyle\int_0^{\pi/2}\frac{\cos x}{\cos x+\sin x}dx=\frac{\pi}{4}$.

16) Show that the parametric form of the straight line which passes the point (a,b) and makes θ angle to the positive direction of an axis, is given by $x=a+t\cos\theta$ and $y=b+t\sin\theta$ (Here t is a parameter)

In the triangle, OAB O lies on the origin B is lying on the first quadrant and OB = 2OA The equations of OA and OB are x - 2y = 0 and 2x +y = 0 respectively. If AB, passes through the point (5,1) show that the co-ordinates of A and B are $(5,\frac{5}{2}),(5,-10)$ and

(5,-10). Find the equation of the circle where the points $(5,\frac{5}{2})$ and (5, – 10) are two end of the diameter. Find the equation of the circles which bisects the circumference of the above circle and passes through the point (0,0) is in the form of $S+\lambda u=0$ (Where a λ is a parameter)

17. i) Find the Solution of $\sin 4\theta \cos 2\theta = \sin 5\theta \cos 3\theta$ for $o \le \theta < \pi/2$

ii) In the usual notation, state the Sine Rule for a triangle.

ABC is a triangle such that b > c. The exterior bisector of the angle CAB intersects produced CB at D. Using the since Rule for the triangles ABC, ACD and ADB.

Show that $\frac{BC}{CD} = \frac{c}{b}$ and deduce that

$$BD = \frac{ac}{b-c} \text{ and } AD = \frac{2bc}{b-c} \sin\left(\frac{A}{2}\right)$$

iii) If $\cos 2x - \sin 2x = R.\cos\left[2x + \frac{\pi}{4}\right]$ for all x, show that $R = \sqrt{2}$.

Sketch the graph of $y = \cos 2x - \sin 2x$ for $-\frac{3\pi}{8} \le x \le \frac{7\pi}{8}$